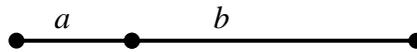


# The Golden Ratio in Garden Design

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## Background

The golden ratio occurs when the ratio of one part to a second part is the same as the second part to the whole.



$a$  and  $b$  form a golden ratio if  $a : b = b : a + b$

In simple terms we could say that the ratio of the smaller part to the bigger part is the same as the ratio of the bigger part to the whole.

This ratio is of real interest occurring in many structures in nature. It also occurs in some apparently diverse areas of mathematics.

We can calculate the relationship algebraically as follows, with the simple solution of a quadratic equation.

$$\frac{a}{b} = \frac{b}{a+b}$$
$$a^2 + ab = b^2$$
$$a^2 + ab - b^2 = 0$$
$$a = \frac{-b \pm \sqrt{b^2 + 4b^2}}{2} = \frac{-b + \sqrt{5}b}{2}, (a > 0)$$
$$\frac{a}{b} = \frac{-1 + \sqrt{5}}{2} = 0.6180 \quad \text{and} \quad \frac{b}{a} = 1.6180$$

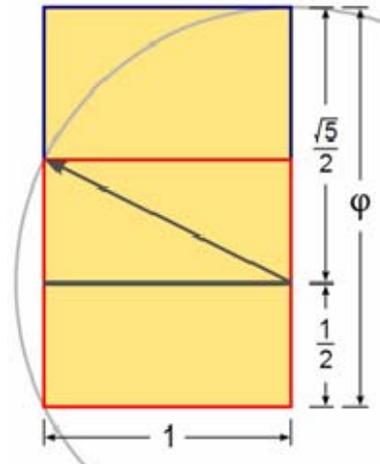
This means that, to 4 decimal places the ratio of  $b : a = 1.6180$  and the ratio of  $a : b = 0.6180$

Historically the golden ratio is known also as the golden section and the ratio of  $b : a$  (or sometimes  $a : b$ ) is known as the golden ratio. This constant is known as phi, or  $\phi$ .

The golden ratio is also significant in the construction of a golden rectangle, which is one in which the sides are in the golden ratio.

The following diagram from Wikipedia shows the construction of a golden rectangle.

Construction of a golden rectangle:  
1. Construct a unit square (red).  
2. Draw a line from the midpoint of one side to an opposite corner.  
3. Use that line as the radius to draw an arc that defines the long dimension of the rectangle.



### The Golden Ratio in history

Different mathematicians through the ages since at least as early as Euclid have expressed an interest in the Golden Ratio. Euclid in his writings referred to the golden ratio although he did not use the terminology. He instead stated that the line AB is divided in extreme and mean ratio by C if  $AB:AC = AC:CB$ . Clearly this relationship has been of interest to mathematicians for hundreds if not thousands of years. The golden mean remains fascinating to mathematicians today.

### Nature and the Golden Ratio

Nature shows a remarkable occurrence of the golden ratio in such areas as seed patterns, leaf arrangements, nautilus shells and flower petals. Mathematician Keith Devlin is one of many when he reports on an explanation for this in his article in his column for the website MAA (Mathematical Association of America) Online in June 2004. He comments that "It's a question of Nature being efficient" and gives plausible explanations about why the golden ratio is an ideal irrational number for this purpose.

### Asthetic Use of the Golden Ratio

The Golden Ratio (sometimes called the Divine Proportion) is considered by many to create shapes which are pleasing visually.

Much has been made of the use of the golden ratio in ancient architecture and art, with many writers presenting what they consider 'clear evidence' of the golden ratio existing in the proportions of, for example, buildings from the Parthenon to the ancient pyramids in Egypt. A wave of opinion fuelled measurement of dimensions of hundreds of pieces of architecture and art to support this belief, and dozens of articles and books have been written about the occurrence of the golden ratio, the golden mean and the divine proportion in different measurements of many famous artistic creations.

The term the 'golden ratio' was in fact only introduced in the first half of the nineteenth century, and it is only during this time period that we have concrete evidence of some artists using the golden ratio as a basis of proportions in their art. Interest prior to this seems to have been mathematical rather than artistic. An article by George Markowsky: *Misconceptions about the Golden ratio* in The College Mathematics Journal Vol 23, January 1992 provides a considered analysis of the use of the golden ratio and it would seem

reasonable to accept that occurrence of values approximately equal to the golden ratio are no more remarkable than the occurrence of other values.

## The Golden Ratio in Japanese Garden Design

David Slawson, in his book *Japanese inspired gardens*, comments on the occurrence of the Golden Section in historic gardens as being seen in the way in which different rocks relate to one another in the 15<sup>th</sup> century design of gardens. It is not clear if there is reference to the golden ratio by the people who designed these gardens.

## The Golden Ratio in Contemporary Design

Beliefs about the beauty of the golden ratio still remain today. Le Corbusier, a Swiss born architect who worked in France in the first half of the twentieth century developed the Modulor, a system of scale and dimension of buildings based on proportions of the human body, the Vitruvian man of da Vinci and the Golden Ratio.

## Computer software

A computer software company [www.Redsofts.com](http://www.Redsofts.com) offers a free piece of software to use for garden design projects. Their website claims that it "allows you to control and correct the sizes and proportions in your design project using the golden section ratio". This allows the designer to create a range of rectangles that conform to the golden ratio by using the mouse and hovering over the drawing. The software can be down loaded from [www.redsofts.com/soft/609/10811/Atrise\\_Golden\\_Section.html](http://www.redsofts.com/soft/609/10811/Atrise_Golden_Section.html). Similar software is offered by software company Atrise (with the note that the software has been downloaded (only) 21 times in the last year.

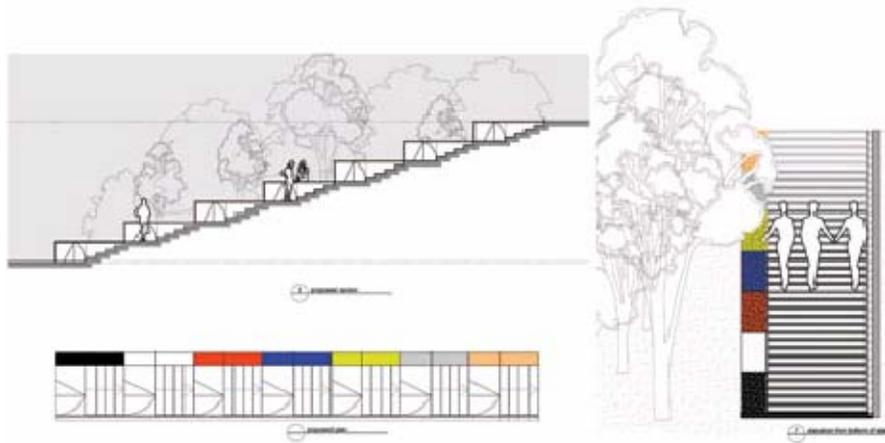
## Designer Lindsay Knapp

Designer Lindsay Knapp who runs a course at the University of Southern Maine Centre for Continuing Education on the use of the Golden Ratio in garden design, explored a very pleasing garden design she had created and looked into the reasons for its success. As a result of her investigation, she attributed it to the fact that, as an artist she appeared to have instinctively in some cases or for other reasons such as the natural dimension of the site, made use of the Golden Ratio and its closely related set of Fibonacci Numbers. The rectangular shape of the site exhibited the proportions of the Golden Rectangle. She found that her groupings of plants were of 1's, 2's, 3's, 5's and 8's. She had placed plants whose heights were in the golden ratio proportion alongside one another. In addition she noted that the flagstone path around the property was laid in groupings of 2, 3 and 5, a subset of the Fibonacci numbers. It would seem that the occurrence of these numbers could be nothing more than coincidence.

[http://www.designtosite.com/knapweed/a\\_garden\\_of\\_golden\\_propor.html](http://www.designtosite.com/knapweed/a_garden_of_golden_propor.html)

## Monkey Hill Reserve

In Auckland one of the local parks this year has been under the process of reconstruction. Monkey Hill Reserve between Onslow and George Streets is being redeveloped according to a design by Mt Eden local resident and internationally-recognised artist, Billy Apple and being implemented in conjunction with a local architectural firm Archifact Ltd. One of the two central design concepts are the Golden Section. The staircase shown below has been designed with its proportions conforming to the Golden Ratio.



### The Battery

In Downtown New York, The Battery is a public area encompassing 25 acres of open space. A master plan has been and continues to be implemented in revitalizing the area, with part of this plan being the reconstruction of the Upper Promenade, including the serpentine Stony Creek granite bench that runs the length of the promenade. Each face of the bench's 23 piers is decorated with a beautifully carved spiral. Named *The Battery Wave*, the spiral is based on the golden mean. The spiral is echoed in the kerbed piers and the arched cast iron supports of the slatted bench. The Remembrance Garden which forms part of the promenade was designed by Piet Oudolf and remembers the victims and families of 9/11.



## Quincy Park

Quincy Park in Cambridge, Massachusetts is a courtyard designed around the concept of the Golden Spiral in 1997 by artist David Phillips.



Spiral shapes and bushes are found in the park along with a rock with a square root symbol on it. There is a plaque with information about the Golden Ratio and the designer. The fence has spirals on the top and there are also sculptures of snails with spiral shells

## Designer Penny Arthurs

Canadian designer Penny Arthurs discusses the harmony and serenity that can be created using geometry, mentioning the golden section as one of the tools that she uses in her work (<http://articles.canadiangardening.com/homepage/default/joys-of-geometry-in-your-garden-n233703p1.html>). She says

“Faced with a sheet of paper marked with little more than the outlines of a plot of land and the house that sits in it, I thank Pythagoras and all who came after him. Using lines taken from the edges, doors and windows of the building, from the boundaries of the property, from the configuration of a stand of trees or anything else close at hand, I can generate shapes-circles, squares, rectangles, triangles-often using the principles of the golden section. These can be divided again and again and connected to each other in a mesh that supports my design.

The shapes don't necessarily appear as such in the final version, but they underpin it. .... Points generated by intersecting lines can be connected into symmetrical or asymmetrical patterns or swooping curves.

Geometry also helps to keep the design coherent and organized, allowing me to align and bring all the bits together in an orderly way and to set up clean axial lines and focal points.

Amazingly, order on the page translates perfectly into order in the garden. There is beauty

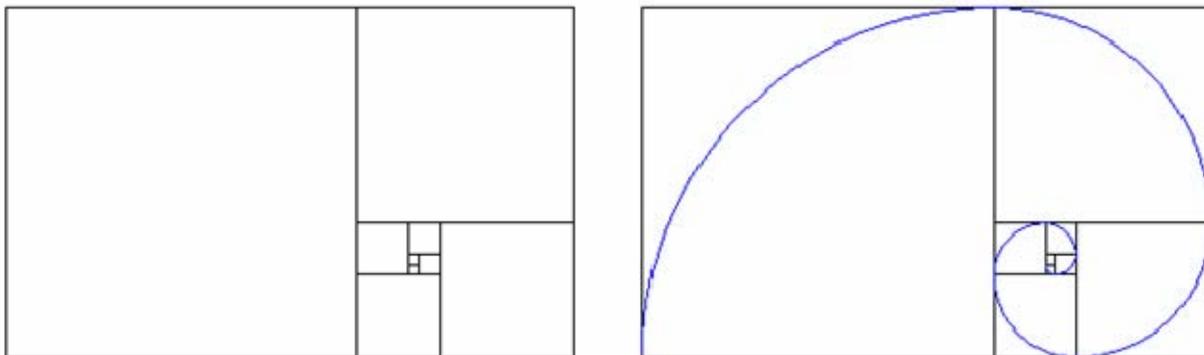
in balance, pleasure in proportion. Plato said, "Geometry existed before creation." That must explain it."

It would seem that her comments encompass more than the golden section and refer to the beauty of geometry more generally.

## The Use of the Golden Spiral in Contemporary Gardens

The golden spiral relates to the Golden Ratio, and can be drawn using the following method using rectangles that conform to the golden ratio in the layout shown below. Quarter circles are then drawn to create the beautiful spiral which is found in nature, for example in the nautilus shell :

The golden spiral is a particularly beautiful spiral, and it is not surprising that garden designers have used this pleasing shape in the design of a number of contemporary gardens.



This Golden Spiral has been used in a number of contemporary gardens.

The College of Engineering at the [California Polytechnic State University](https://www.calpoly.edu/) has created its engineering plaza using the golden ratio in the form of a spiral. Their belief is that the Fibonacci Series spiral is representative of engineering knowledge (and more specifically mathematical knowledge).

